Horava gravity as palladium of locality, unitarity and renormalizability: methods and results

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Plan

Horava gravity:

1) Renormalizability and regular gauges: projectable models

2) BRST structure of renormalization and background formalism

3) Asymptotic freedom in (2+1)-dimensional model

4) Method of universal functional traces in background field formalism

5) Beta functions and RG fixed points in (3+1)-dimensions

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B.,
PRD 93, 064022 (2016), arXiv:1512.02250; arXiv:1705.03480;
PRL 119, 211301 (2017), arXiv:1706.06809;

M. Herrero-Valea, S. Sibiryakov & A.B., PRD100 (2019) 026012;

A.Kurov, S.Sibiryakov & A.B., Phys.Rev.D 105 (2022) 4, 044009
arXiv: 2110.14688

Renormalization of Horava gravity

Saving unitarity in renormalizable QG

Einstein GR $$S_{EH} = \frac{M_P^2}{2} \int dt d^d x \, R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2} \int dt d^d x \, (h_{ij} \Box h_{ij} + h^2 \Box h + \dots)$$

Higher derivative gravity

Stelle (1977)

$$\int (M_P^2 R + R_{\mu\nu} R^{\mu\nu} + R^2)$$

$$\Rightarrow \int (M_P^2 h_{ij} \Box h_{ij} + h_{ij} \Box^2 h_{ij} + \dots)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Horava (2009)

$$\underbrace{\int dt\, d^d x}_{\propto\, b^{-(z+d)}} \big(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \dots\big)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \quad \Longrightarrow \quad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \quad \Longrightarrow \quad \gamma_{ij} \quad N^i, \qquad i = 1, \dots, d$$

$$t \mapsto \tilde{t}(t) \quad \Longrightarrow \quad N$$

Foliation preserving diffeomorphisms

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \ , \quad t \mapsto \tilde{t}(t)$$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \quad i, j = 1, \ldots, d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1, \ [t] = -z, \ [N^i] = z - 1, \ [\gamma_{ij}] = 0, \qquad [K_{ij}] = z.$$

extrinsic curvature

$$\boxed{K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)}$$

``Projectable" theory $N = \text{const} = 1$

Horava gravity action

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\textit{kinetic term -- unitarity}} - \mathcal{V}(\gamma) \Big)$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$
$$+\nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \dots$$

Many more versions: extra structures in non-projectable theory, reduction of structures for detailed balance case . . .

$$N \neq \text{const}, \quad a_i = \nabla_i \ln N, \dots$$

$d + 1 = 4$ **DoF: tt-graviton and scalar**

Unitarity domain (no ghosts) $\frac{1-\lambda}{1-3\lambda} > 0$

$$\omega_{tt}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \ ,$$
$$\omega_s^2 = \frac{1-\lambda}{1-3\lambda}\Big(- \eta k^2 + (8\mu_1 + 3\mu_2) k^4 + (8\nu_4 + 3\nu_5) k^6 \Big)$$

tachyon in IR

Divergences power counting

$$\text{Deg of div} \int \frac{d^{d+1}p}{\left(p^2\right)^N} = d + 1 - 2N = \text{physical dimensionality}$$

$$\Downarrow \qquad p = (\omega, \mathbf{k}), \quad p^2 \to \omega^2 + \mathbf{k}^{2z}$$

$$\text{Deg of div} \int \frac{d\omega\, d^d k}{\left(\omega^2 + \mathbf{k}^{2z}\right)^N} = z + d - 2zN = \text{scaling dimensionality}$$

$$\text{physical dimensionality} \neq \text{scaling dimensionality}$$

$$\boxed{z = d}$$

critical value

$$\mathcal{V}^{(d=2)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2$$

$$\mathcal{V}^{(d=3)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}^2 + O(R^3, R\nabla^2 R)$$

Things are not so simple: power counting is not enough:

$$\int \prod_{l=1}^{L} d^{d+1}k^{(l)}\ \mathcal{F}_n(k) \prod_{m=1}^{M} \frac{1}{\left(P^{(m)}(k)\right)^2} \Rightarrow \int \prod_{l=1}^{L} d\omega^{(l)} d^d k^{(l)}\ \mathcal{F}_n(\omega, \mathrm{k}) \prod_{m=1}^{M} \frac{1}{A_m\left(\Omega^{(m)}(\omega)\right)^2 + B_m\left(\mathrm{K}^{(m)}(\mathrm{k})\right)^{2z}}$$

Generalization of BPHZ renormalization theory (subtraction of subdivergences) works only for $\underbrace{A_m > 0 \text{ and } B_m > 0}_{\text{depends on gauge fixing}}$

Invention of regular gauges for projectable HG

$$F^\mu \equiv \partial_\nu h^{\nu\mu} + \ldots \ \Rightarrow\ F^i = \dot{N}^i + c\partial_j \Delta^{d-1} h^{ji} + \ldots$$

extra derivatives to have homogeneity in scaling

$$[\,F^i\,] = 2d - 1 \ \Rightarrow\ \mathcal{O}_{ij} = \Delta^{2-d}(\Delta\delta_{ij} + \xi\partial_i\partial_j)^{-1}$$

Gauge fixing term $$S_{\mathrm{gf}} = \frac{\sigma}{2G}\int dt\, d^d x\, \sqrt{\gamma}\, F_i \mathcal{O}^{ij} F_j$$

σ, ξ *free gauge fixing parameters*

$$\Downarrow$$

Projectable HG is renormalizable in any *d*

Gauge invariance of counterterms

Background covariant gauge conditions + BRST structure of renormalization

Background field method:

$$\gamma_{ij} = \bar\gamma_{ij} + h_{ij},\ \ \partial_i \to \bar\nabla_i,\ \ \mathcal{O}^{-1\,ij} = \bar\Delta^{d-1}\,\bar\gamma^{ij} + \xi\bar\nabla^i\Delta^{d-2}\bar\nabla^j,\ \ldots$$

DeWitt, Tyutin, Voronov, Stelle, Batalin, Vilkovisky, Slavnov, Arefieva, Abbott…
Barnich, Henneaux, Grassi, Anselmi,…

Blas, Herrero-Valea, Sibiryakov, Steinwachs & A.B.
arXiv: 1705.03480, JHEP07(2018)035

Background field extension of the BRST operator + inclusion of generating functional sources into the gauge fermion

$$\Downarrow$$

1. *BRST structure of renormalization via **decoupling of the background field***
2. ***No** power counting or use of field dimensionalities*
3. *Extension to **Lorentz symmetry** violating theories*
4. *Extension to (nonrenormalizable) **effective field theories***

Extended BRS operator and gauge fermion $\quad Q \to Q_{\rm ext}, \quad \varPsi \to \varPsi_{\rm ext}[\,\varPhi, J\,]$

$$e^{-W/\hbar} = \int D\varPhi\, e^{-(S+Q\varPsi+J\varPhi)/\hbar} \quad \Longrightarrow \quad e^{-W/\hbar} = \int D\varPhi\, e^{-(S+Q_{\rm ext}\varPsi_{\rm ext})/\hbar}$$

Gauge independence on shell $\quad \boxed{\delta_\varPsi W[\,J\,]\Big|_{J=0} = 0}$

Renormalization:

$$S[\,\varphi\,] \to S[\,\varphi\,] + \Delta_\infty S[\,\varphi\,]$$

physical gauge invariant local counterterm

$$\boldsymbol{\varPsi}_{\rm ext}[\,\varPhi\,] \to \boldsymbol{\varPsi}_{\rm ext}[\,\varPhi\,] + \Delta_\infty \boldsymbol{\varPsi}_{\rm ext}[\,\varPhi\,]$$

*local counterterm to gauge fermion **(irrelevant)***

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G} \int dt\, d^2x\; N\sqrt{\gamma}\; \left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathrm{loop}} \to \Gamma_{1-\mathrm{loop}} + \int dt\, d^dx\, \Omega_{ij} \frac{\delta S}{\delta \gamma_{ij}}$$

Essential coupling constants:

$$\lambda, \quad \mathcal{G} \equiv \frac{G}{\sqrt{\mu}}$$

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma} \mathcal{O}^{-1}_{ij} (\nabla^k h^j_k - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij} \Delta + \xi \nabla_i \nabla_j]^{-1}$$

background covariant gauge-fixing term

σ, ξ – free parameters

$$\Downarrow$$

Mathematica package xAct

D. Brizuela, J. M. Martin-Garcia, and G. A. Mena Marugan, Gen. Rel. Grav. 41, 2415 (2009), arXiv:0807.0824

$$\beta_\lambda = \frac{15 - 14\lambda}{64\pi} \sqrt{\frac{1 - 2\lambda}{1 - \lambda}}\, \mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi (1 - \lambda)^2} \sqrt{\frac{1 - \lambda}{1 - 2\lambda}}\, \mathcal{G}^2$$

Powerful check – gauge independence of essential couplings

Check in conformal gauge $h_{ij} = e^{2\phi}\gamma_{ij}$

$$\beta_\mu = \frac{2 - 7\lambda + 6\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda)}} G\sqrt{\mu}$$

$$\beta_G = -\frac{6\lambda - 7}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$$\Longrightarrow \beta_{\mathcal{G}}$$

Compare to regular ``relativistic'' gauge

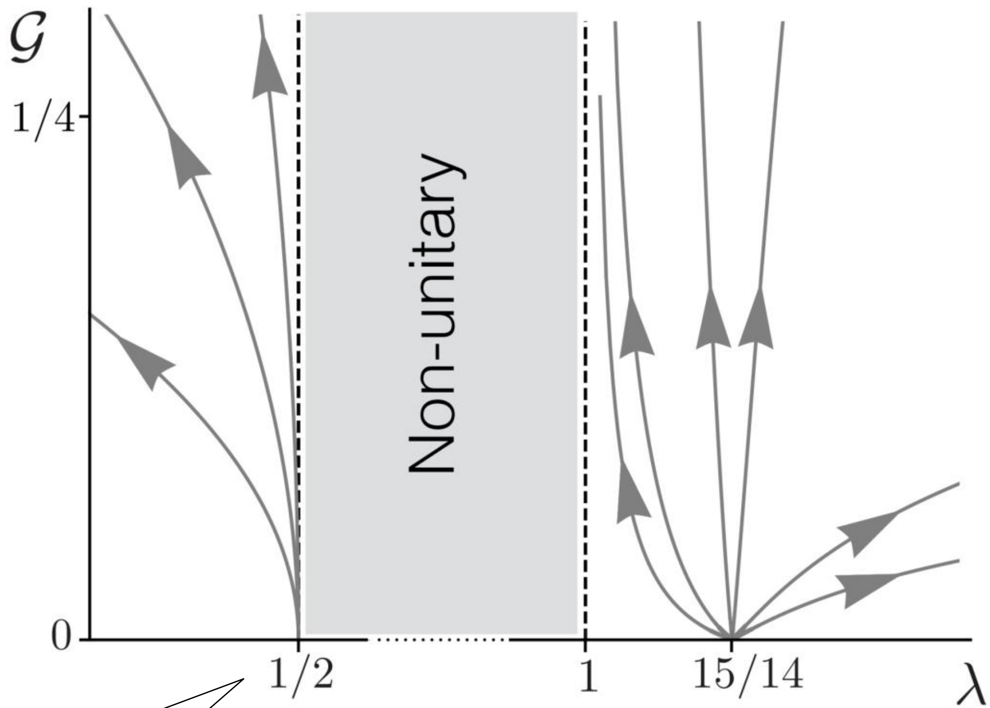
$$\beta_\mu = \frac{30 - 73\lambda + 42\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda)}} G\sqrt{\mu}$$

$$\beta_G = -\frac{30\lambda - 23}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$$\Longrightarrow \beta_{\mathcal{G}}$$

same

Renormalization flows:



strongly coupled fixed point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2\lambda)^2}{64\pi(1-\lambda)^{3/2}}\,\tilde{\mathcal{G}}^2$$

AF UV fixed point

Towards RG flows of (3+1)-dimensional Horava gravity

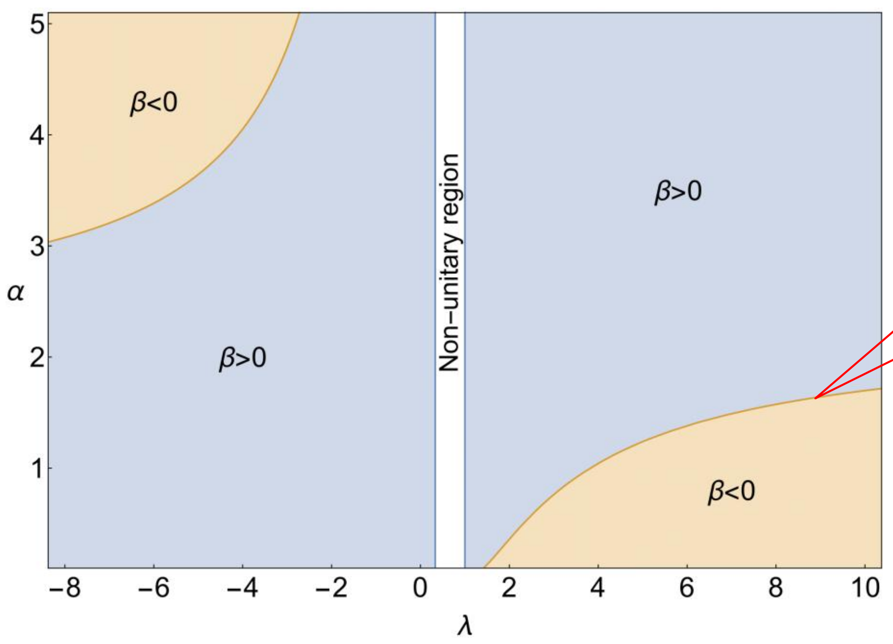
$$S=\frac{1}{2G}\int dt\,d^dx\sqrt{\gamma}\Big(K_{ij}K^{ij}-\lambda K^2-\mathcal{V}(\gamma)\Big)$$

$$\mathcal{V}(\gamma)=\nu_1R^3+\nu_2RR_{ij}R^{ij}+\nu_3R^i_jR^j_kR^k_i+\nu_4\nabla_iR\nabla^iR+\nu_5\nabla_iR_{jk}\nabla^iR^{jk}+\ldots$$

$$\boxed{\beta_G,\qquad \beta_\lambda}$$

M. Herrero-Valea, S. Sibiryakov & A.B.,
PRD100 (2019) 026012

$$\alpha\equiv\frac{1}{\nu_5}\frac{(1-\lambda)(8\nu_4+3\nu_5)}{1-3\lambda}$$



$$\lambda(\alpha)=\frac{9+7\sqrt{\alpha}-2\alpha+2\sqrt{10(\alpha+\alpha^{3/2})}}{3(3+\sqrt{\alpha}-2\alpha)}$$

The lines $\lambda=\lambda(\alpha)$ correspond to potential location of fixed points of the full RG flow. The region $\lambda\in[1/3,1]$ is excluded by the requirement of unitarity.

Background field method

One-loop effective action

$$\Gamma_{\text{one-loop}} = \frac{1}{2}\mathrm{Tr}_4 \ln \hat{F}(\nabla) = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{-s\hat{F}(\nabla)}$$

Action Hessian $\hat{F}(\nabla) = F^A_B(\nabla)$ *acting in the space of fields* $\varphi = \varphi^A(x)$

$$\hat{F}(\nabla) = \Box + \hat{P} - \frac{\hat{1}}{6}R, \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$[\nabla_\mu, \nabla_\nu]\, V^\lambda = R^\lambda{}_{\rho\mu\nu} V^\rho, \quad [\nabla_\mu, \nabla_\nu]\, \phi = \hat{\boldsymbol{R}}_{\mu\nu}\phi, \quad [\nabla_\mu, \nabla_\nu]\, \hat{X} = [\hat{\boldsymbol{R}}_{\mu\nu}, \hat{X}]$$

Heat kernel (Schwinger-DeWitt) expansion

$$e^{-s\hat{F}(\nabla)}\delta(x,y) = \frac{\mathcal{D}^{1/2}(x,y)}{(4\pi s)^{d/2}}\, g^{1/2}(y)\, e^{-\frac{\sigma(x,y)}{2s}} \sum_{n=0}^{\infty} s^n\, \hat{a}_n(x,y)$$

Schwinger-DeWitt (Gilkey-Seely) coefficients

$$\hat{a}_0\Big|_{y=x} = \hat{1}, \quad \hat{a}_1\Big|_{y=x} = \hat{P},$$

$$\hat{a}_2\Big|_{y=x} = \frac{1}{180}\left(R^2_{\alpha\beta\gamma\delta} - R^2_{\mu\nu} + \Box R\right)\hat{1} + \frac{1}{12}\hat{\boldsymbol{R}}^2_{\mu\nu} + \frac{1}{2}\hat{P}^2 + \frac{1}{6}\Box\hat{P}, \ldots$$

One-loop divergences

$$\Gamma^{\text{div}}_{\text{one-loop}} = -\frac{1}{32\pi^2\varepsilon}\int dx\, g^{1/2}\mathrm{tr}\,\hat{a}_2(x,x), \quad \varepsilon = 2 - \frac{d}{2} \to 0$$

Extension to non-minimal and higher-derivative operators

The method of universal functional traces (I. Jack and H. Osborn (1984), G.A.Vilkovisky & A.B., Phys. Rept. 119 (1985) 1)

$$\mathrm{Tr}\,\ln\big(\Box^N + P(\nabla)\big) = N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,\ln\Big(1 + P(\nabla)\frac{1}{\Box^N}\Big)$$
$$= N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,P(\nabla)\frac{1}{\Box^N} + \cdots$$

$$\Gamma^{\mathrm{div}} = \sum_{m,n}\int d^4x\,\mathcal{R}_n^{\mu_1\ldots\mu_m}\underbrace{\nabla_{\mu_1}\ldots\nabla_{\mu_m}\frac{\hat{1}}{\Box^n}\delta(x,y)\,\Big|_{y=x}^{\mathrm{div}}}_{\text{universal functional traces}}$$

universal functional traces

$$\nabla\ldots\nabla\frac{\hat{1}}{\Box^n}\,\delta(x,y)\,\Big|_{y=x}^{\mathrm{div}} = \frac{(-1)^n}{\Gamma(n)}\nabla\ldots\nabla\int_0^\infty ds\,s^{\alpha-1}\,\underbrace{e^{s\Box}\,\hat{\delta}(x,y)}_{\text{Schwinger-DeWitt expansion}}\,\Big|_{y=x}^{\mathrm{div}}$$

Schwinger-DeWitt expansion

Horava gravity:

$$\varphi^A(x) = h_{ij}(x), n^i(x) \;\; + \;\; \text{FP ghosts}$$

Static 3-metric background with generic 3-metric $\bar{\gamma}_{ij}(\mathbf{x})$

$$\gamma_{ij}(x) = \bar{\gamma}_{ij}(\mathbf{x}) + h_{ij}(\tau, \mathbf{x}), \qquad N^i(x) = 0 + n^i(\tau, \mathbf{x})$$

Hessian structure $\hat{F}(\nabla) = -\hat{1}\,\partial_\tau^2 + \hat{\mathbb{F}}(\nabla_{\mathbf{x}})$

Space parts of metric and vector (shifts and ghosts) operators:

$$\hat{\mathbb{F}} = \mathbb{F}^A_{\;B} = \left\{ \mathbb{F}_{ij}^{\;\;kl}, \mathbb{F}^k_{\;i} \right\} \sim \nabla^6 + \ldots$$

Example – for the ghost operator in (σ,ξ) -family of gauges:

$$\mathbb{F}^i_{\;j}(\nabla) = -\frac{1}{2\sigma}\delta^i_j \Delta^3 - \frac{1}{2\sigma}\Delta^2\nabla_j\nabla^i - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla^k\nabla_j\nabla_k$$
$$-\frac{\xi}{2\sigma}\nabla^i\Delta\nabla_j\Delta + \frac{\lambda}{\sigma}\Delta^2\nabla^i\nabla_j + \frac{\lambda\xi}{\sigma}\nabla^i\Delta^2\nabla_j, \qquad \Delta = \gamma^{ij}\nabla_i\nabla_j$$

Dimensional reduction method on a static background with generic 3-metric

$$\mathrm{Tr}_4 \ln(-\partial_\tau^2 + \mathbb{F}) = -\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}_4 e^{-s(-\partial_\tau^2+\mathbb{F})}$$
$$= \int d\tau\, d^3x \int \frac{ds}{s}\,\mathrm{tr}\, e^{-s(-\partial_\tau^2+\mathbb{F})}\delta(\tau-\tau')\,\delta(\mathbf{x}-\mathbf{x}')\Big|_{\tau=\tau',\,\mathbf{x}=\mathbf{x}'}$$
$$= -\int d\tau\ \mathrm{Tr}_3\sqrt{\mathbb{F}}$$

How to proceed with the square root of the 6-th order differential operator?

$$\mathbb{F} = \sum_{a=0}^{6} \mathcal{R}_{(a)} \sum_{6\geq 2k\geq a} \alpha_{a,k}\nabla_1\ldots\nabla_{2k-a}(-\Delta)^{3-k}, \quad \mathcal{R}_{(a)} = O\Big(\frac{1}{l^a}\Big)$$

Pseudodifferential operator – infinite series in curvature invariants $\mathcal{R}_{(a)}$

$$\sqrt{\mathbb{F}} = \sum_{a=0}^{\infty} \mathcal{R}_{(a)} \sum_{k\geq a/2}^{K_a} \tilde{\alpha}_{a,k}\nabla_1\ldots\nabla_{2k-a}\frac{1}{(-\Delta)^{k-3/2}}$$

How to find the coefficients $\tilde{\alpha}_{a,k}$?

$$\sqrt{\mathbb{F}} = \mathbb{Q}^{(0)} + \mathbb{X}$$

$$\textit{principal symbol of } \mathbb{F} \equiv \mathbb{F}(\nabla)\Big|_{\nabla \to p,\, \mathcal{R} \to 0}$$

$$\mathbb{Q}^{(0)} = \Big(\textit{principal symbol of } \mathbb{F}\Big)^{1/2}\Big|_{p \to \nabla}$$

Solving by iterations the linear equation for $\mathbb{X}$ as expansion in the curvature

$$\mathbb{Q}^{(0)}\mathbb{X} + \mathbb{X}\,\mathbb{Q}^{(0)} = \mathbb{F} - \Big(\mathbb{Q}^{(0)}\Big)^2 - \mathbb{X}^2 \;\propto \mathcal{R} \sim [\nabla, \nabla]$$

$$\mathrm{Tr}_3\sqrt{\mathbb{F}}\,\Big|^{\mathrm{div}} = \sum_{a=2}^{6}\sum_{k}\tilde{\alpha}_{a,k}\int d^3x\,\mathcal{R}_{(a)}(\mathbf{x})\nabla_1...\nabla_{2k-a}\frac{1}{(-\Delta)^{k-3/2}}\delta(\mathbf{x},\mathbf{x}')\,\Big|^{\mathrm{div}}_{\mathbf{x}=\mathbf{x}'}$$

Divergences of universal functional traces

$$\nabla...\nabla\frac{\hat{1}}{(-\Delta)^{\alpha}}\delta(x,y)\,\Big|^{\mathrm{div}}_{y=x} = \frac{1}{\Gamma(\alpha)}\nabla...\nabla\int_0^\infty ds\, s^{\alpha-1}\, e^{s\Delta}\,\hat{\delta}(x,y)\,\Big|^{\mathrm{div}}_{y=x}$$

Schwinger-DeWitt expansion

Examples:

$$g^{ij}(-\Delta)^{1/2}\delta_{ij}{}^{kl}(x,y)\,\Big|^{\mathrm{div}}_{y=x} = -\frac{1}{16\pi^2\varepsilon}\sqrt{g}\,g^{kl}\frac{1}{30}\left(\frac{1}{2}R_{ij}^2+\frac{1}{4}R^2+\Delta R\right)$$

$$\int d^3x\,\delta_{kl}{}^{ij}(-\Delta)^{3/2}\delta_{ij}{}^{kl}(x,y)\,\Big|^{\mathrm{div}}_{y=x} = \frac{3}{32\pi^2\varepsilon}\int d^3x\,\sqrt{g}\,\delta_{kl}{}^{ij}\,a_{3\,ij}{}^{kl}(x,x)$$

$$= \frac{3}{32\pi^2\varepsilon}\int d^3x\,\sqrt{g}\,\left(\frac{31}{45}R^i_jR^j_kR^k_i-\frac{233}{210}R_{ij}^2R+\frac{673}{2520}R^3+\frac{5}{84}R\Delta R-\frac{67}{420}R_{ij}\Delta R^{ij}\right)$$

Results for beta functions of (3+1)-dimensional Horava gravity

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Six essential coupling constants $\mathcal{G},\ \lambda$ and $\chi = (u_s, v_1, v_2, v_3)$

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \quad \lambda, \quad u_s = \sqrt{\frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{(1-3\lambda)\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1,2,3$$

$$\beta_{\mathcal{G}} = \frac{\mathcal{G}^2}{26880\pi^2(1-\lambda)^2(1-3\lambda)^2(1+u_s)^3 u_s^3} \sum_{n=0}^{7} u_s^n \mathcal{P}_n^{\mathcal{G}}[l, v_1, v_2, v_3]$$

$$\beta_\lambda = \frac{\mathcal{G}}{120\pi^2(1-\lambda)(1+u_s)u_s}\Big[27(1-\lambda)^2 + 3u_s(11-3\lambda)(1-\lambda) - 2u_s^2(1-3\lambda)^2\Big]$$

$$\beta_\chi = \frac{A_\chi \mathcal{G}}{26880\pi^2(1-\lambda)^3(1-3\lambda)^3(1+u_s)^3 u_s^5} \sum_{n=0}^{9} u_s^n \mathcal{P}_n^{\chi}[l, v_1, v_2, v_3]$$

$$A_{u_s} = u_s(1-\lambda), \quad A_{v_1} = 1, \quad A_{v_2} = A_{v_3} = 2$$

$\mathcal{P}_n^\chi[l, v_1, v_2, v_3,]$ *are polynomials in* λ *and* v_a,

Use of Mathematica package xAct

Example (one of the longest ones):

$$\begin{aligned}\mathcal{P}_5^{v_1}=&-2(1-\lambda)^2(1-3\lambda)\Big\{168v_2^3(51\lambda^3-149\lambda^2+125\lambda-27)-108v_3^3(9\lambda^3+9\lambda^2\\&-25\lambda+7)-4v_2^2(1-\lambda)\Big[18v_3(117\lambda^2-366\lambda+109)-284\lambda^2-7265\lambda+5425\Big]\\&+40320v_1^2(1-\lambda)^2(\lambda+1)-9v_3^2(3467\lambda^3-8839\lambda^2+6237\lambda-865)\\&+v_1\Big[64v_2^2(1-\lambda)^2(1717\lambda-581)-16v_2(1-\lambda)\Big(3v_3(2741\lambda^2-3690\lambda+949)\\&+25940\lambda^2-40662\lambda+12022\Big)+27v_3^2(961\lambda^3-2395\lambda^2+1835\lambda-401)\\&+6v_3(52267\lambda^3-148963\lambda^2+129881\lambda-33185)-288353\lambda^3+542255\lambda^2\\&-333355\lambda+83485\Big]-2v_2\Big[162v_3^2(3\lambda^3+35\lambda^2-51\lambda+13)+24v_3(1265\lambda^3\\&-2191\lambda^2+691\lambda+235)+30971\lambda^3-40323\lambda^2+13167\lambda-4451\Big]-12v_3(6551\lambda^3\\&-11593\lambda^2+6124\lambda-1112)+109519\lambda^3-252396\lambda^2+177357\lambda-34396\Big\}\end{aligned}$$

Check of the results: independence of essential beta functions on the choice of gauge ($\sigma\,\xi$- family of gauge conditions) and spectral sum method in dimensional and zeta-functional regularization.

Discussion: detailed balance and asymptotic freedom

Special (not fully fixed) point: $\{v^*\} : v_1 = 1/2, \quad v_2 = -5/2, \quad v_3 = 3$

$$\beta_{v_a}\Big|_{\{v^*\},\, u_s \to 0} = 0\ ,\ a = 1,2,3\ , \qquad \beta_{u_s}\Big|_{\{v^*\},\, u_s \to 0}, \beta_{\mathcal{G}}\Big|_{\{v^*\},\, u_s \to 0} \text{ are regular}$$

Detailed balance version of HG

$$S_{v^*, u_s \to 0} = \frac{1}{2G}\int d\tau\, d^3x\, \sqrt{\gamma}\, (K_{ij}K^{ij} - \lambda K^2 + \nu_5\, C^{ij}C_{ij})$$
$$= \frac{2}{G}\int d\tau\, d^3x\, \sqrt{\gamma}\, (K_{ij} + \sqrt{\nu_5} C_{ij})\, \mathbb{G}^{ij,kl}\, (K_{kl} + \sqrt{\nu_5} C_{kl}),$$
$$C^{ij} = \varepsilon^{ikl}\nabla_k\Big(R^j_l - \frac{1}{4}\, R\, \delta^j_l\Big) = \varepsilon^{kl(i}\nabla_k R^{j)}_l$$

Connection to gravitational Chern-Simons theory

$$C^{ij} = -\frac{1}{\sqrt{g}}\frac{\delta W_{\mathrm{CS}}[g]}{\delta g_{ij}(x)}, \qquad W_{\mathrm{CS}}[g] = \frac{1}{2}\int d^3x\, \epsilon^{ijk}\Big(\Gamma^m_{il}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\Big)$$

$\mathcal{G} \to 0$ *asymptotic freedom*

$\mathcal{G} \to \infty$ *Landau pole*

Fixed points equations:

$$\beta_\lambda / \mathcal{G} = 0 \,,$$

$$\beta_\chi / \mathcal{G} = 0 \,, \qquad \chi = u_s, v_1, v_2, v_3$$

λ	u_s	v_1	v_2	v_3	$\beta_\mathcal{G}/\mathcal{G}^2$	AF?	UV attractive along λ?
0.1787	60.57	-928.4	-6.206	-1.711	-0.1416	yes	no
0.2773	390.6	-19.88	-12.45	2.341	-0.2180	yes	no
0.3288	54533	3.798×10^8	-48.66	4.736	-0.8484	yes	no
0.3289	57317	-4.125×10^8	-49.17	4.734	-0.8784	yes	no

Special limit: $\lambda \to \infty$ ***(non-relativistic gravity vs Perelman-Ricci flow, A. Frenkel, P. Horava and S. Randall, 2011.1914; cosmology implication, [A.E. Gumrukcuoglu](), [S. Mukohyama,]() 1104.2087)***

u_s	v_1	v_2	v_3	$\beta_{\mathcal{G}}/\mathcal{G}^2$	asymptotically free?	UV attractive along λ?
0.01950	0.4994	-2.498	2.999	-0.2004	yes	no
0.04180	-0.01237	-0.4204	1.321	-1.144	yes	no
0.05530	-0.2266	0.4136	0.7177	-1.079	yes	no
12.28	-215.1	-6.007	-2.210	-0.1267	yes	yes
21.60	-17.22	-11.43	1.855	-0.1936	yes	yes
440.4	-13566	-2.467	2.967	0.05822	no	yes
571.9	-9.401	13.50	-18.25	-0.07454	yes	yes
950.6	-61.35	11.86	3.064	0.4237	no	yes

Conclusions

Renormalization of Horava-Lifshitz gravity

Salvation of unitarity in local renormalizable QG via LI violation

BPHZ renormalization and "regularity" of propagators

Gauge invariance of UV counterterms

Asymptotic freedom in (2+1)-dimensional theory

Method of universal functional traces

Beta functions of (3+1)-dimensional theory and fixed points candidates for AF

THANK YOU!